

# THEORY OF PROBABILITY AND MATHEMATICAL STATISTICS 

## Syllabus

|  | Requisites of the Course |
| :--- | :--- |
| Cycle of Higher Education | First cycle of higher education (Bachelor's degree) |
| Field of Study | 12 Information Technologies |
| Speciality | 123 Computer Engineering |
| Education Program | Computer Systems and Networks |
| Type of Course | Normative |
| Mode of Studies | full-time |
| Year of studies, semester | 2 year (3 semester) |
| ECTS workload | 5 credits (ECTS)/150 hours |
| Testing and assessment | 3 semester - Final test |
| Course Schedule | 3 classes per week by the timetable http://rozklad.kpi.ua/ |
| Language of Instruction | English |
|  | Lecturer: PhD, Associate Professor, Oleksandr Markovskyi, mobile <br> $+380967108534, ~ e m a i l ~ m a r k o v s k y y @ i . u a, ~ p e r s o n a l ~ w e b ~ p a g e ~$ |
| https://bbb.comsys.kpi.ua/b/ole-qgu-7rw |  |$|$| Teacher of practical work: PhD, Associate Professor, Oleksandr Markovskyi, |
| :--- |
| Course Instructors |
| mobile +380967108534, email markovskyy@i.ua, personal web page |
| https://bbb.comsys.kpi.ua/b/ole-qgu-7rw |

Access to the course

## Outline of the Course

1. Course description, goals, objectives, and learning outcomes

The role of discipline in the training of specialists in computer engineering, computer systems and networks is determined by the fact that methods of probability theory and mathematical statistics are an important tool for developing computer hardware and software, an important factor in their effectiveness for a wide range of practical applications.

The value of the discipline is determined by the need for specialists in computer systems and networks in theoretical knowledge of probability theory and mathematical statistics and practical skills in applying methods and techniques for solving problems of analysis of probabilistic situations and random variables.

The purpose of teaching the discipline is to reveal modern scientific concepts, concepts of probability theory and mathematical statistics, methods of analysis of random events, quantities and processes, as well as methods of statistical processing of experimental data.
"Is part of the cycle of natural science training and is part of the theoretical basis of a set of knowledge and skills that form a specialist in the field of computer systems and networks.

As a result of studying the discipline the student must:
Know:

- Basic axioms and theorems of probability theory;
- Mathematical models for working with random events, random discrete and continuous quantities and their systems, as well as random processes;
- Basic theoretical provisions of mathematical statistics, methods for assessing the reliability of measurement results, hypothesis testing, decision making, correlation and regression analysis;
- Basic theoretical provisions of the theory of random processes, flows, Markov chains and processes, queuing systems,
- Basic provisions of information theory and its coding.

Be able to:

- independently analyze random events, calculate their probability;
- analyze discrete and continuous random variables, as well as their systems, calculate probabilistic characteristics of quantities with different distribution laws, independent and interconnected;
- apply methods of probability theory to calculate the performance and reliability of computer systems and their components;
- process the results of measurements, obtain estimates of the characteristics of random variables and their systems, determine estimates of the reliability of measurements, perform correlation and regression analysis;
- independently assess the conformity of measurement results to the theoretical distribution, analyze the statistical homogeneity of several measurement results;
- apply decision-making methods in conditions of statistical uncertainty; random processes to analyze the operation of computer systems;
- to determine the parameters of the transient process and stationary characteristics of discrete and continuous random Markov processes;
- to independently apply the mathematical apparatus of queuing theory to analyze the characteristics of information processing processes in computer systems and networks; \{1\}\} - to determine the quantitative characteristics of information, to optimize the coding of data taking into account their information characteristics,
- to independently develop programs for solving scientific and technical calculations related to random events, quantities and processes, generation programs random variables with given distribution characteristics.

3К1, ЗК3, ПРН1, ПРН2, ПРН6.
2. Prerequisites and post-requisites of the course (the place of the course in the scheme of studies in accordance with curriculum)

## Prerequisites courses

Higher mathematics
Programming
Discrete mathematics
English (basic level A2)

## Post-requisites course

System programming
Algorithms and calculation methods
Organization of computing processes
Computer networks
Reliability of computer systems
Protection of information in computer systems
Computer modeling

## 3. Content of the course

## Module 1. Probability theory

1.1. Basic concepts of probability theory. Random events. Classical, geometric and statistical ways to determine the probability of events. Basic axioms of probability theory.
1.2. The formula of total probability, features of its application in practice. A posteriori probability and its definition using the Bayesian formula.
1.3. Model of word tests, Bernoulli's formula, features of its application, formulas of approximate
calculations. Poisson's formula.
1.4. The concept of discrete and continuous random variables. The concept of distribution laws, the main numerical characteristics of random variables. Binomial distribution.
1.5. Continuous random variables. Distribution density, integral distribution function. Numerical characteristics of continuous random variables. Uniform distribution law. Exponential distribution law, its application to assess the reliability of computer system components.
1.6. Normal distribution law, calculation of probabilities using Laplace functions, determination of distribution parameters for a given probability. Central limit theorem. Muavra-Laplace theorem and its application for calculating probabilities in binomial distribution.
1.7. Systems of random variables. Dependent and independent random variables. Covariance and correlation - characteristics of the degree of dependence of quantities. Method for calculating probabilities for dependent random variables.
1.8. Functions of random variables. Distribution of sum, difference and product of random variables.
1.9. Basic concepts of information theory. Determining the amount of information and entropy. Optimization of data coding from the standpoint of information theory.

## Module 2.Mathematical statistics

2.1. Basic concepts, definitions and problems of mathematical statistics. Determining the characteristics of the sample. Confidence probability and confidence interval for estimates of mathematical expectation, variance and correlation, methods of their determination. Theorem on the error of the arithmetic mean and its application in measuring techniques
2.2. Estimation of reliability of hypotheses concerning the law of distribution of a random variable. Neumann-Pearson method. Methods for assessing the validity of the hypothesis of homogeneity of two samples.
2.3.Correlation and regression analysis. Linear and nonlinear regression. technique of reproduction of linear regression dependence for many variables. Estimation of regression error.
2.4. Methods of decision making in conditions of uncertainty. Errors of decision making of the first and second kind. Decision-making according to the criteria of error limit value and according to the Neumann-Pearson method. Comparative analysis of methods and their choice for application.
2.5. Basic concepts and problems of the theory of random processes. Basic characteristics and classification of random processes. Flows of events, their types, characteristics and methods of their definition.
2.7. Elements of queuing theory. Formulas for determining the main characteristics: the average length of the queue, the delay time in the system and their variances. Queuing systems with arbitrary distribution of service time. Queuing systems with priorities
2.8. Discrete and continuous random Markov processes. Determining the characteristics of the transient process and methods of calculating stationary probabilities. Non-ergodic Markov processes and methods for determining their characteristics.Application of Markov process apparatus for analysis of computational processes.

## 4. Coursebooks and teaching resources

Free E-book and guides on Probability Theory whose can be downloaded

1. Niels Richard Hansen. Probability Theory and Statistics Lecture notes.
2. Scott Sheffield. Theory of Probability.
3. Manuel Cabral Morais. Lecture Notes Probability Theory
4. E. T. Jaynes. Probability Theory The Logic of Science
5. MJ Evans. Probability and Statistics

Additional teaching resources
6. Achim Klenke. Probability Theory. A Comprehensiv Course. 2020
7. Daniel W. Stroock. Probability Theory. An Analytic View. 2010
8. Michael A. Proschan, Pamela A. Shaw Essentials of Probability Theory for Statisticians

## Educational content

## 5. Methodology

Practical classes should give students the skills to solve practical problems of software development, hardware, and analysis of the effectiveness of computer tools using probability theory.

In the first practical session, students gain the skills to determine the probability of an event. In practice, the use of probability theory in computer technology to determine the probability of an event is often based on the classical model.

To use this model successfully, you need to have combinatorics skills.
Combinatorics. as a science that provides methods for solving problems:

- calculating the number of options:
- organizing the search for options

This is important for the successful work of a programmer
Formally, there are many formulas combinatorics, but for their application it is necessary to form combinatorial thinking. The main elements in the study of combinatorics are mastering the formula of inclusion-exclusion, the principles of combinatorics of choice, and combinatorics of grouping.

The classical formula for determining the probability of an event can be generalized for cases where the set of possible events contains an infinite number, but for this set there is a measure S . Then we can determine the measure $S$ of a subset of events of interest.

Length, area or volume are most often used as a measure.
The probabilities of complex events are determined on the basis of mathematical transformations over the probabilities of simple events. This uses the axioms of probability theory, as well as models of complex events, the most common of which is the Bernoulli model.

For diagnostic tasks in computer technology, the concept of a priori probability and the formula for its calculation play an important role.

Discrete random variables are given by the distribution law. The law of distribution of a discrete random variable for each of its values compares the probability that the random variable is equal to this value.

Continuous random variables can take an infinite number of values. The probability that a continuous value of $X$ takes a specific value is zero. We can only determine the probability that a random variable $X$ takes values within certain limits. The main characteristic of a continuous random variable x is the law of its distribution, which is given by the distribution density function. Accordingly, the definition of probabilities is reduced to the calculation of integrals from the distribution density function.

In computer technology, random variables are the most important. In practice, to calculate the probabilities of these random variables, you need to have the skills to use Laplace tables or numerical integration.

The ability to determine the numerical characteristics of random variables plays an important role. To solve this problem, you must also implement the calculation of integrals.

For computer technicians, the ability to generate random events and random variables with a given software distribution law plays an important role.

Exponential distribution theory is widely used to evaluate the reliability of computer software and hardware.

## 6. Self-study

The main forms of Self-study of students in studying the course Probability Theory and Mathematical Statistics are:

- Self- study of auxiliary theoretical material, which is required for successful mastering of basic knowledge and skills of their practical use. In particular, the basic provisions of combinatorics are presented for self study. To ensure self-study of this section is organized: a special consultation in the 2th week of stading $\backslash$, giving students a set of standard tasks in combinatorics, checking the tasks and issuing, if necessary, individual tasks.
- Self-study solution by students of published tasks with further verification by the teacher. Targeted individual consultations are conducted based on the results of independent problem solving.
- Self execution of research works and projects in which methods of probability theory and mathematical statistics are used to solve problems of development and analysis of computer technologies.


## Policy and Assessment

## 7. Course policy

- The student must, in the absence of good reasons, attend lectures. The student has the right to ask questions and receive a comprehensive answer. The teacher has the right to record the student's presence at lectures.
- The student must, for no good reason, attend practical classes. In these classes, the student has the right to ask questions and receive a comprehensive answer. The teacher has the right to record the presence of the student in practical classes, has the right to ask questions to the student, to involve him in solving problems.


## 8. Monitoring and grading policy

At the first class the students are acquainted with the grading policy which is based on Regulations on the system of assessment of learning outcomes https://document.kpi.ua/files/2020 1273.pdf The student's rating in the course consists of points that he/she receives for participation in 27 practical classes (R1), based on the results of two modular tests (R2) and a final test (R3).

## Rs=R1+R2+R3=100 points

Points can be earned on practical classes for solving tasks. The maximum average weight score on practical classes is equal 60 points.

## 10 points $\times 2$ module test $=20$ points

## Final test $=\mathbf{2 0}$ points

According to the university regulations on the monitoring of students' academic progress (https://kpi.ua/document control) there are two assessment weeks, usually during 7th/8th and 14th/15th week of the semester, when students take the Progress and Module tests respectively, to
check their progress against the criteria of the course assessment policy.
The students who finally score the required number of points ( $\geq 60$ ) can:

- get their final grade according to the rating score;
- perform a Fail/Pass test in order to increase the grade.

Students whose final performance score is below 60 points but more than 30 are required to complete a Fail/Pass test. If the grade for the test is lower than the grade, which the student gets for his semester activity, a strict requirement is applied - the student's previous rating is canceled and he receives a grade based on the results of the Fail/Pass test. Students whose score is below 30 are not allowed to take the Fail/ Pass Test.

The final performance score or the results of the Fail/ Pass Test are adopted by university grading system as follows:

| Score | Grade |
| :---: | :---: |
| $100-95$ | Excellent |
| $94-85$ | Very good |
| $84-75$ | Good |
| $74-65$ | Satisfactory |
| $64-60$ | Sufficient |
| Below 60 | Fail |
| Course requirements are not met | Not Graded |

## 9. Additional information about the course

## Syllabus of the course

Is designed by teacher PhD, Associate Professor, Oleksandr Markovskyi
Adopted by Department of Computing Technics (protocol № 10 , 25052022 )
Approved by the Faculty Board of Methodology (protocol № 10 , 0906 2022)

